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Fluid Flow in Petroleum Reservoirs
Syllabus and Administrative Procedures
Fall 2013

Al-Farabi Kazakh National University
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Required Texts/Resources:

1. *Advanced Mathematics for Engineers and Scientists*, M.R. Spiegel, Schaum's Series (1971).
2. *Handbook of Mathematical Functions*, M. Abramowitz and I. Stegun, Dover Pub. (1972).
3. *Table of Laplace Transforms*, G.E. Roberts and H. Kaufman, W.B. Saunder, Co. (1964).
4. *Numerical Methods*, R.W. Hornbeck, Quantum Publishers, Inc., New York (1975).
5. *Approximations for Digital Computers*: Hastings, C., Jr., *et al*, Princeton U. Press, Princeton, New Jersey (1955).
6. *Handbook for Computing Elementary Functions*: L.A. Lyusternik, *et al*, Pergamon Press, (1965).

Optional Texts/Resources:

1. *Calculus*, 4th edition: Frank Ayres and Elliot Mendelson, Schaum's Outline Series (1999).
2. *Differential Equations*, 2nd edition: Richard Bronson, Schaum's Outline Series (1994).
3. *Conduction of Heat in Solids*, 2nd edition, H. Carslaw and J. Jaeger, Oxford Science Publications (1959).
4. *Laplace Transforms*, M.R. Spiegel, Schaum's Outline Series (1965) — Local bookstores.
5. *Numerical Analysis*, F. Scheid, Schaum's Outline Series, McGraw-Hill Book Co, New York (1968). — Local bookstores.
6. *Methods of Numerical Integration*, P.F. Davis and P. Rabinowitz, Academic Press, New York (1989).
7. *Table of Integrals, Series, and Products*, I.S. Gradshteyn and I.M. Ryzhik, Academic Press (1980).
8. *An Atlas of Functions*, J. Spanier and K. Oldham, Hemisphere Publishing (1987).
9. *The Mathematics of Diffusion*, 2nd edition, J. Crank, Oxford Science Publications (1975).
10. *Adv. Mathematical Methods for Eng. and Scientists*, 2nd edition, C.M. Bender and S.A. Orsag, McGraw-Hill (1978).
11. *Asymptotic Approximations of Integrals*, R. Wong, Academic Press (1989).
12. *Asymptotics and Special Functions*, F.W.J. Olver, Academic Press (1974).

Basis for Grade:

Homeworks/Projects	90 percent
Class Participation	10 percent
	Total = 100 percent

Course Description

Graduate Catalog: Analysis of fluid flow in bounded and unbounded reservoirs, wellbore storage, phase redistribution, finite and infinite conductivity vertical fractures, dual-porosity systems.

Translation: Development of skills required to derive "classic" problems in reservoir engineering and well testing from the fundamental principles of mathematics and physics. Emphasis is placed on a mastery of fundamental calculus, analytical and numerical solutions of 1st and 2nd order ordinary and partial differential equations, as well as extensions to non-linear partial differential equations that arise for the flow of fluids in porous media.

Fluid Flow in Petroleum Reservoirs
Course Objectives
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Course Objectives

The student should be able to demonstrate mastery of objectives in the following areas:

- **Module 1 — Advanced Mathematics Relevant to Problems in Engineering**
- **Module 2 — Petrophysical Properties**
- **Module 3 — Fundamentals of Flow in Porous Media**
- **Module 4 — Reservoir Flow Solutions**
- **Module 5 — Applications/Extensions of Reservoir Flow Solutions**

Considering these modular topics, we have the following catalog of course objectives:

Module 1: Advanced Mathematics Relevant to Problems in Engineering

- **Fundamental Topics in Mathematics:**
  - Work fundamental problems in algebra and trigonometry, including partial fractions and the factoring of equations.
  - Perform elementary and advanced calculus: analytical integration and differentiation of elementary functions (polynomials, exponentials, and logarithms), trigonometric functions (sin, cos, tan, sinh, cosh, tanh, and combinations), and special functions (Error, Gamma, Exponential Integral, and Bessel functions).
  - Derive the Taylor series expansions and Chebyshev economizations for a given function.
  - Derive and apply formulas for the numerical differentiation and integration of a function using Taylor series expansions. Specifically, be able to derive the forward, backward, and central "finite-difference" relations for differentiation, as well as the "Trapezoidal" and "Simpson's" Rules for integration.
  - Apply the Gaussian and Laguerre quadrature formulas for numerical integration.
- **Solution of First and Second Order Ordinary Differential Equations:**
  - First Order Ordinary Differential Equations:
    — Classify the order of a differential equation (order of the highest derivative).
    — Verify a given solution of a differential equation via substitution of a given solution into the original differential equation.
    — Solve first order ordinary differential equations using the method of separation of variables (or separable equations).
    — Derive the *method of integrating factors* for a first order ordinary differential equation.
    — Apply the Euler and Runge-Kutta methods to numerically solve first order ordinary differential equations.
  - Second Order Ordinary Differential Equations:
    — Develop the *homogeneous* (or *complementary*) solution of a 2nd order ordinary differential equation (ODE) using $y=e^{mx}$ as a trial solution.
    — Develop the *particular* solution of a 2nd order ordinary differential equation (ODE) using the method of undetermined coefficients.
    — Apply the Runge-Kutta method to numerically solve second order ordinary differential equations.
- **The Laplace Transform:**
  - Fundamentals of the Laplace Transform:
    — State the definition of the Laplace transformation and its inverse.
    — Derive the operational theorems for the Laplace transform.
    — Demonstrate familiarity with the "unit step" function.
    — Develop and apply the Laplace transform formulas for the discrete data functions

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Course Objectives (Continued)

Module 1: Advanced Mathematics Relevant to Problems in Engineering (continued)

- **The Laplace Transform: (continued)**
  - Applications of the Laplace Transform to Solve Linear Ordinary Differential Equations:
    — Develop the Laplace transform of a given differential equation and its initial condition(s).
    — Resolve the algebra resulting from taking the Laplace transform of a given differential equation and its initial condition(s) into a closed and hopefully, invertible form.
    — Invert the closed form Laplace transform solution of a given differential equation using the properties of Laplace transforms, Laplace transform tables, partial fractions, and prayer.
  - Numerical Laplace Transform and Inversion:
    — Use the Gauss-Laguerre integration formula for numerical Laplace transformation.
    — Demonstrate familiarity with the development of the Gaver formula for the numerical inversion of Laplace transforms.
    — Apply the Gaver and Gaver-Stehfest numerical Laplace transform inversion algorithms.
- **Special Functions:**
  - Demonstrate familiarity with and be able to apply the following "special functions:"
    — Exponential Integral (Ei (x) and $E_1(x)$= -Ei $(-x)$).
    — Gamma and Incomplete Gamma Functions ($\Gamma(x)$, and $\gamma(a,x)$, $\Gamma(a,x)$ and $B(z,w)$).
    — Error and Complimentary Error Functions (erf(x) and erfc(x)).
    — Bessel Functions: $J_0(x)$, $J_1(x)$, $Y_0(x)$, and $Y_1(x)$.
    — Modified Bessel Functions: $I_0(x)$, $I_1(x)$, $K_0(x)$, and $K_1(x)$, and integrals of $I_0(x)$, $K_0(x)$.

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Course Objectives (Continued)

Module 2: Petrophysical Properties

- **Porosity and Permeability Concepts:**
  - Be able to recognize and classify rock types:
    — Clastics (sandstones) and Carbonates (limestones, chalks, dolstones), and
    — Be familiar with the porosity and permeability characteristics of these rocks.
  - Be familiar with factors that affect porosity. In particular, the shapes, arrangements, and distributions of grain particles and the effect of cementation, vugs, and fractures on porosity.
  - Be familiar with correlative relations for porosity and permeability.
  - Be familiar with "friction factor/Reynolds Number" concept put forth by Cornell and Katz for flow through porous media. Be aware that this plotting concept validates Darcy's law empirically (the unit slope line on the left portion of the plot, laminar flow).

- **Correlation of Petrophysical Data:**
  - Be familiar with the various models for permeability based on porosity, grain size sorting parameters, irreducible water saturation, electrical and surface area parameters, nuclear magnetic resonance parameters, etc. as described by Nelson[1] (*The Log Analyst* (May-June 1994), 38-62).

- **Concept of Permeability—Darcy's Law:**
  - Development of Darcy's Law for fluid flow in porous media via analogy with the Poiseuille equation for laminar fluid flow in pipes. Be able to develop a velocity/pressure gradient relation for modelling the flow of fluids in pipes (*i.e.*, the Poiseuille equation--given below).

    $v_{avg} = \dfrac{q}{A_x} = k_p \dfrac{1}{\mu} \dfrac{\Delta p}{\Delta x}$ where $k_p = \dfrac{r^2}{8}$ is considered to be a "geometry" factor.
  - Units Conversions:
    — Be able to derive the "units" of a Darcy (1 Darcy = 9.86923×10^{-9} cm^2).
    — Be able to derive the field and SI unit forms of Darcy's law.

- **Capillary Pressure:**
  - Be familiar with the concept of "capillary pressure" for tubes as well as for porous media—and be able to derive the capillary pressure relation for fluid rise in a tube.
  - Be familiar with and be able to derive the Purcell-Burdine permeability and relative permeability relations for porous media using the "bundle of capillary tubes" model as provided by Nakornthap and Evans (Nakornthap, K. and Evans, R.D.: "Temperature-Dependent Relative Permeability and Its Effect on Oil Displacement by Thermal Methods," *SPERE* (May 1986) 230-242.).
  - Be familiar with and be able to derive the Brooks-Corey-Burdine equation for permeability based on the Purcell-Burdine permeability equation (Brooks, R.H. and Corey, A.T.: "Properties of Porous Media Affecting Fluid Flow," *J. Irrigation and Drainage Division Proc.*, ASCE (1966) **92**, No. IR 2, 61.).

- **Relative Permeability:**
  - Be familiar with the concept of "relative permeability" and the factors that should and should not affect this function. You should also be familiar with the laboratory techniques for measuring relative permeability.
  - Be familiar with and be able to derive the Purcell-Burdine relative permeability equations.
  - Be familiar with and be able to derive the Brooks-Corey-Burdine equations for relative permeability.

- **Electrical Properties of Reservoir Rocks:**
  - Be familiar with the definition of the formation resistivity factor, F, as well as the effects of reservoir and fluid properties on this parameter.
  - Be familiar with and be able to use the Archie and Humble equations to estimate porosity given the formation resistivity factor, F.
  - Be familiar with the definition of the resistivity index, I, as well as the effects of reservoir and fluid properties on this parameter and also be familiar with the Archie result for water saturation, S_w.
  - Be familiar with the "shaly sand" models given by Waxman and Smits for relating the resistivity index with saturation and for relating formation factor with porosity.

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Course Objectives (Continued)

Module 3: Fundamentals of Flow in Porous Media

- **Steady-State Flow Concepts: Laminar Flow**
  - Derive the concept of permeability (Darcy's Law) using the analogy of the Poiseuille equation for the flow of fluids in capillaries. Be able to derive the "units" of a "Darcy" (1 Darcy = 9.86923×10^{-9} cm^2), and be able to derive Darcy's Law in "field" and "SI" units.
  - Derive the single-phase, steady-state flow relations for the laminar flow of gases and compressible liquids using Darcy's Law — in terms of pressure, pressure-squared, and pseudopressure, as appropriate.
  - Derive the steady-state "skin factor" relations for radial flow.
- **Steady-State Flow Concepts: Non-Laminar Flow**
  - Demonstrate familiarity with the concept of "gas slippage" as defined by Klinkenberg.
  - Derive the single-phase, steady-state flow relations for the non-laminar flow of gases and compressible liquids using the Forchheimer equation (quadratic in velocity) — in terms of pressure, pressure-squared, and pseudopressure, as appropriate.
- **Material Balance Concepts:**
  - Be able to identify/apply material balance relations for gas and compressible liquid systems.
  - Be familiar with and be able to apply the "Havlena-Odeh" formulations of the oil and gas material balance equations.
- **Pseudosteady-State Flow Concepts:**
  - Demonstrate familiarity with and be able to derive the single-phase, pseudosteady-state flow relations for the laminar flow of compressible liquids in a radial flow system (given the radial diffusivity equation as a starting point).
  - Sketch the pressure distributions during steady-state and pseudosteady-state flow conditions in a radial system.
- **Development of the Diffusivity Equation for Flow in Porous Media:**
  - Derive the following relations for single-phase flow: (general flow geometry)
    - — The pseudopressure/pseudotime forms of the diffusivity equation for cases where fluid density and viscosity are and are not functions of pressure.
    - — The diffusivity equations for oil and gas cases in terms B_o or B_g.
    - — The diffusivity equation for the flow of a "slightly compressible liquid.
    - — The diffusivity equation for gas flow in terms of pressure and p/z.
    - — The diffusivity equations for single-phase gas flow in terms of the following: pseudopressure, pressure-squared, and pressure — using the "general" approach in each case (*i.e.*, starting with the p/z formulation).
  - Derive the following relations for multiphase flow: (general flow geometry)
    - — The continuity relations for the oil, gas, and water phases in terms of the fluid densities, also be able to "convert" the density form of the continuity equation to the formation volume factor form.
    - — The mass accumulation and mass flux relations for the oil, gas, and water phases in terms of the fluid formation volume factors.
    - — The Martin relations for total compressibility and the associated saturation-pressure relations (Martin Eqs. 10 and 11). Be able to show all details.

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Course Objectives (Continued)

Module 4: Reservoir Flow Solutions

- **Dimensionless Variables:**
  - Develop the dimensionless form of the single-phase radial flow diffusivity equation as well as the appropriate dimensionless forms of the initial and boundary conditions, including the developments of dimensionless radius, pressure, and time.
  - Derive the conversion factors for dimensionless pressure and time, for SI and "field" units.
- **Radial Flow Solutions:**
  - Derive the real domain (time) solution for the constant rate inner boundary condition and the infinite-acting reservoir outer boundary condition using both the Laplace transform and the Boltzmann transform approaches. Also be able to derive the "log-approximation" for this solution.
  - Derive the general and particular solutions (in the Laplace domain) for a well produced at a constant flow rate in a radial homogeneous reservoir for the following conditions:
    - — Initial Condition: Uniform Pressure Distribution
    - — Inner Boundary Condition: Constant Flowrate at the Well
    - — Outer Boundary Conditions: Prescribed Flux or Constant Pressure at the Boundary
- **Linear Flow Solutions:**
  - Derive the general and particular solutions (in the Laplace domain) for a well produced at a constant flow rate in a linear homogeneous reservoir for the following conditions:
    - — Initial Condition: Uniform Pressure Distribution
    - — Inner Boundary Condition: Constant Flowrate at the Well
    - — Outer Boundary Conditions: Infinite-Acting Reservoir Condition—or a Prescribed Flux or Constant Pressure at the Boundary
- **Vertically Fractured Wells:**
  - Demonstrate familiarity with the concept of a well with a uniform flux or infinite conductivity vertical fracture in a homogeneous reservoir. Note that the uniform flux condition implies that the rate of fluid entering the fracture is constant at any point along the fracture. On the other hand, for the infinite conductivity case, we assume that there is *no pressure drop* in the fracture as fluid flows from the fracture tip to the well.
  - Derive the real and Laplace domain (line source) solutions for a well with a uniform flux or infinite conductivity vertical fracture in a homogeneous reservoir.
- **Dual Porosity/Naturally Fractured Reservoirs: (Warren and Root Approach— Pseudosteady-State Interporosity Flow)**
  - Show familiarity with the "fracture" and "matrix" models developed by Warren and Root.
  - Derive the Laplace and real domain results (by Warren and Root) for pseudosteady-state interporosity flow.
- **Solution of the Non-Linear Radial Flow Gas Diffusivity Equation**
  - Demonstrate familiarity with the convolution form of a non-linear partial differential equation (*i.e.*, a p.d.e. with a non-linear right-hand-side term).
  - Derive the generalized Laplace domain formulation of the non-linear radial gas diffusivity equation using the "convolution" approach.
- **Convolution and Wellbore Storage**
  - Derive the convolution sums and integrals for the variable-rate and variable pressure drop cases, and be able to derive the real and Laplace domain identities for relating the constant pressure and constant rate cases (from van Everdingen and Hurst).
  - Derive the relations which model the phenomena of "wellbore storage," based on physical principles (*i.e.*, material balance)

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Course Objectives (Continued)

Module 4: Reservoir Flow Solutions — *Under Construction/Consideration*

- **Multilayered Reservoir Solutions**
- **Dual Permeability Reservoir Solutions**
- **Horizontal Well Solutions**
- **Radial Composite Reservoir Solutions**
- **Various Models for Flow Impediment (Skin Factor)**

Module 5: Applications/Extensions of Reservoir Flow Solutions — *Under Construction/Consideration*

- **Oil and Gas Well Flow Solutions for Analysis, Interpretation, and Prediction of Well Performance.**
- **Low Permeability/Heterogeneous Reservoir Behavior.**
- **Macro-Level Thermodynamics (coupling PVT behavior with Reservoir Flow Solutions).**
- **External Drive Mechanisms (Water Influx/Water Drive, Well Interference, etc.).**
- **Hydraulic Fracturing/Solutions for Fractured Well Behavior.**
- **Analytical/Numerical Solutions of Various Reservoir Flow Problems.**
- **Applied Reservoir Engineering Solutions — Material Balance, Flow Solutions, etc.**

Fluid Flow in Petroleum Reservoirs
Course Outline
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Topic

Module 1: Advanced Mathematics Relevant to Problems in Engineering

- Review of Fundamentals and Introduction to Calculus
- Approximation of Functions
  — Taylor Series Expansions and Chebyshev Economizations
  — Numerical Differentiation and Integration of Analytic Functions and Applications
  — Least Squares
- First-Order Ordinary Differential Equations
- Second-Order Ordinary Differential Equations
- The Laplace Transform
  — Fundamentals of the Laplace Transform
  — Properties of the Laplace Transform
  — Applications of the Laplace Transform to Solve Linear Ordinary Differential Eqs.
  — Numerical Laplace Transform and Inversion
- Introduction to Special Functions

Module 2: Petrophysical Properties

- Porosity and Permeability Concepts
- Correlation of Petrophysical Data
- Concept of Permeability — Darcy's Law
- Capillary Pressure
- Relative Permeability
- Electrical Properties of Reservoir Rocks

Module 3: Fundamentals of Flow in Porous Media

- Steady-State Flow Concepts: Laminar Flow
- Steady-State Flow Concepts: Non-Laminar Flow
- Material Balance Concepts
- Pseudosteady-State Flow in a Circular Reservoir
- Development of the Diffusivity Equation for Liquid Flow
- Development of the Diffusivity Equations for Gas Flow
- Development of the Diffusivity Equation for Multiphase Flow

Module 4: Reservoir Flow Solutions (**Under Construction/Consideration*)

- Dimensionless Variables and the Dimensionless Radial Flow Diffusivity Equation
- Solutions of the Radial Flow Diffusivity Equation — Infinite-Acting Reservoir Case
- Laplace Transform (Radial Flow) Solutions — Bounded Circular Reservoir Cases
- Real Domain (Radial Flow) Solutions — Bounded Circular Reservoir Cases
- Linear Flow Solutions: Infinite and Finite-Acting Reservoir Cases
- Solutions for a Fractured Well — High Fracture Conductivity Cases
- Dual Porosity Reservoirs — Pseudosteady-State Interporosity Flow Behavior
- Direct Solution of the Gas Diffusivity Equation Using Laplace Transform Methods
- Convolution and Concepts and Applications in Wellbore Storage Distortion
- Multilayered Reservoir Solutions and/or Dual Permeability Reservoir Solutions*
- Horizontal Well Solutions*
- Radial Composite Reservoir Solutions and/or Models for Flow Impediment (Skin Factor)*

Module 5: Applications/Extensions of Reservoir Flow Solutions (**Under Construction/Consideration*)

- Oil and Gas Well Flow Solutions for Analysis, Interpretation, and Prediction of Well Performance*
- Low Permeability/Heterogeneous Reservoir Behavior*
- Macro-Level Thermodynamics (coupling PVT behavior with Reservoir Flow Solutions)*
- Hydraulic Fracturing/Solutions for Fractured Well Behavior*
- Applied Reservoir Engineering Solutions — Material Balance, Flow Solutions, etc.*

Fluid Flow in Petroleum Reservoirs
Homework Topics and Format Guidelines
Fall 2013

Homework Topics: (These are intended topics, addition and/or deletion of certain problems may occur as other problems become available. Multiple assignments from each topic are likely.)

- Review of algebra and fundamental mathematics.
- Analytical and numerical problems in calculus.
- Laplace transform methods — analytical and computational considerations.
- Solution of ordinary differential equations.
- Special functions — analytical and computational considerations.
- Development of steady-state flow equations from physical principles.
- Development of pseudosteady-state flow equations from the diffusivity equation.
- Development of "diffusion" equations from physical principles.
- Solution of diffusion-type partial differential equations.
- Development and application of various well/reservoir/production solutions.

Computing Topics: Students will be asked to make numerical computations for certain problems — in such cases the student will generally be allowed to select the computational product for their work.

Homework Format Guidelines:

I. **General Instructions:** *You must use engineering analysis paper or lined notebook paper, and this paper must measure 8.5 inches in width by 11 inches in height*

1. *You must only write on the front of the page!*
2. Number all pages in the upper right-hand corner and staple all pages together in upper left-hand corner. You must also put your name (or initials) in the upper right corner of each page next to the page number.
3. Place the following identification on a cover page: (*Do not fold*)

Name:	(printed)
Date:	Day-Month-Year
Assignment:	(Specific)

II. **Outline of Homework Format**

1. Given: (Base Data)
2. Required: (Problem Objectives)
3. Solution: (Methodology)
   A. Sketches and Diagrams
   B. Assumption, Working Hypotheses, References
   C. Formulas and Definitions of Symbols (Including Units)
   D. Calculations (Including Units)
4. Results
5. Conclusions: Provide a short summary that discusses the problem results.

III. **Guidelines for Paper Reviews**

For each paper you are to address the following questions: (Type or write neatly)

- Problem:
  — What is/are the problem(s) solved?
  — What are the underlying physical principles used in the solution(s)?
- Assumptions and Limitations:
  — What are the assumptions and limitations of the solutions/results?
  — How serious are these assumptions and limitations?
- Practical Applications:
  — What are the practical applications of the solutions/results?
  — If there are no obvious "practical" applications, then how *could* the solutions/results be used in practice?
- Discussion:
  — Discuss the author(s)'s view of the solutions/results.
  — Discuss your own view of the solutions/results.
- Recommendations/Extensions:
  — How could the solutions/results be extended or improved?
  — Are there applications other than those given by the author(s) where the solution(s) or the concepts used in the solution(s) could be applied?

Fluid Flow in Petroleum Reservoirs
General Advice for Study and Class Preparation
Fall 2013

Faculty-Student Contract:
The most important element of your education is your participation. No matter how hard we as faculty try (or don't try) to prepare you to learn, we cannot force you to work. We can only provide examples of how you should perform and we can only evaluate your performance — not your intentions or your personality, nor can we make allowances for your personal problems or your lack of preparation.

We can of course provide some pretty unpleasant alternatives as incentives (*e.g.*, poor grades), but poor grades are a product of only two issues, a lack of subject mastery, or apathy. We as faculty can do much to prepare you for a rewarding career, not only as engineers, but also as productive members of society in whatever capacity you wish to serve. But—we cannot make you care, we cannot make you prepare, and we cannot make you perform — only you can do this.

We have chosen our path in life to help you find yours, we want you to succeed (perhaps sometimes more than you do) and we will do our best to make your education fulfilling and rewarding. As we embark on what will likely be a tedious and challenging experience, we reaffirm our commitment to seeing that you get the most out of your education. When it seems as though we are overbearing taskmasters (and we may well be), remember that we are trying to prepare you for challenges where there is no safety net — and where there may be no second chance.

Our goal is to be your guide — we will treat you with the respect and consideration that you deserve, but you must have the faith to follow, the dedication to prepare, and the determination to succeed — it will be your turn to lead soon enough.

General Procedures for Studying:

1. Before each lecture you should read the text carefully, don't just scan topics, but try to resolve sections of the reading into a simple summary of two or three sentences, emphasizing concepts as well as methods.
2. During the lecture take careful notes of what your instructor says and writes, LISTEN to what is being said as well as how it is emphasized. Don't try to be neat, but do try to get every detail you can — think of the lecture as an important story that you will have to tell again later.
3. As soon as possible after the lecture (and certainly the same day), reread the text and your "messy" lecture notes, then rewrite your lecture notes in a clear and neat format — redrawing the figures, filling in missed steps, and reworking examples. You are probably thinking that no one in their right mind would do this—but the secret is that successful students always review and prepare well in advance of exams.
4. Prepare a list of questions or issues that you need clarified, ask your instructor at the start of the next class (so others can benefit) or if you need one-on-one help, see your instructor as soon as possible, do not assume that it will "come to you later."
5. Work one homework problem at a time, without rushing. You are not learning if you are rushing, copying, or scribbling. Spread the problems out in time and write down any questions you have.
6. ASK QUESTIONS. In class, during office hours, ANY chance you get. If you do not understand something you cannot use it to solve problems. It will not come to you by magic. ASK! ASK! ASK!
7. Practice working problems. In addition to assigned problems, work the unassigned ones. Where do you think faculty take exam questions? You should establish a study group and distribute the load — but you should work several of each type of problem that you are assigned.
8. Before a test, you should go over the material covered by preparing an outline of the important material from your notes as well as the text. Then rewrite your outline for the material about which you are not very confident. Review that material, then rewrite the notes for the material about which you are still not confident. Continue until you think that you understand ALL of the material.
9. "Looking over" isn't learning, reading someone else's solution is insufficient to develop your skills, you must prepare in earnest — work lots and lots of problems, old homework, old exams, and study guide questions.
10. Speed on exams is often critical. It is not just a test of what you know, but how well you know it (and how fast you show it). The point is not just to "understand" but to "get it in your bones."
11. Participate in class. The instructor must have feedback to help you. Force the issue if you must, it is your education.